Factor demand derivation from the firm's cost function

Consider the following cost function:

$$C = x \cdot \left(w + 2\sqrt{wr} + r\right)$$

Derive the factor demand functions, knowing that w is the price of L, r is the price of K, and x is the quantity of output

Solution

Since the cost function is given, we can obtain the conditional factor demands by applying Shephard's lemma

$$L(w,r,x) = \frac{\partial C(w,r,x)}{\partial w} \qquad K(w,r,x) = \frac{\partial C(w,r,x)}{\partial r}$$

We first compute the derivative with respect to w

$$C(w,r,x) = x\left(w + 2\sqrt{wr} + r\right)$$

$$\frac{\partial C}{\partial w} = x\left(1 + 2 \cdot \frac{1}{2}(wr)^{-1/2} r\right)$$

$$\frac{\partial C}{\partial w} = x\left(1 + \frac{r}{\sqrt{wr}}\right)$$

$$\frac{\partial C}{\partial w} = x\left(1 + \sqrt{\frac{r}{w}}\right)$$

Hence, the labor demand is

$$L(w,r,x) = x\left(1 + \sqrt{\frac{r}{w}}\right)$$

Now we compute the derivative with respect to r

$$\frac{\partial C}{\partial r} = x\left(1 + 2 \cdot \frac{1}{2}(wr)^{-1/2} w\right)$$

$$\frac{\partial C}{\partial r} = x\left(1 + \frac{w}{\sqrt{wr}}\right)$$

$$\frac{\partial C}{\partial r} = x\left(1 + \sqrt{\frac{w}{r}}\right)$$

Hence, the capital demand is

$$K(w,r,x) = x\left(1 + \sqrt{\frac{w}{r}}\right)$$

Therefore, the conditional factor demand functions are

$$L(w,r,x) = x\left(1 + \sqrt{\frac{r}{w}}\right) \qquad K(w,r,x) = x\left(1 + \sqrt{\frac{w}{r}}\right)$$